Observable core response in neutron star spin glitches

Bennett Link
Montana State University, Department of Physics, Bozeman MT 59717[*]
(Dated: November 12, 2012)

The superfluid core of a neutron star is usually assumed to corotate with the crust over timescales longer than minutes. I show that the interaction between the neutron superfluid and the type II superconductor of the outer core increases the coupling time to weeks or longer. I suggest that observed post-glitch response over timescales of weeks to years represents recovery of the outer core. Spin glitches could originate in either the inner crust or the outer core.

PACS numbers: 97.60.Jd, 26.60.-c, 47.37.+q, 97.60.Gb

Introduction. The problem of the origin of spin glitches in neutron stars (NSs) has remained unsolved since the first glitch was observed in the Vela pulsar in 1969. This problem is of considerable interest, as an understanding of the glitch phenomenon would offer insights into both the dynamical and ground-state properties of matter above nuclear density. To make progress on the glitch problem, it is crucial to identify which components of the liquid interior corotate with the crust, store and release the angular momentum that drives glitches, and produce the observed post-glitch response that occurs over days to years (see [1] for examples).

The outer core of a NS was predicted to contain superfluid neutrons and superconducting protons long ago ([2]; for a recent review, see [3]). The neutron flow around the vortices that thread the rotating superfluid entrains a proton mass current, giving the vortices very strong magnetization [4]. Electrons scatter efficiently with the magnetized vortices, enforcing corotation between the neutron fluid and the proton-electron fluid over a timescale of $400 - 10^4 P$, where P is the rotational period in seconds [5]. For typical rotation periods of < 0.1 s, the coupling time is minutes. Corotation between the charged core fluid and the crust is maintained by a combination of Ekman pumping and magnetic stresses [6]. The entire core is thus usually assumed to corotate with the crust. If this assumption is correct, glitches can arise only in the inner-crust superfluid that coexists with the solid lattice, where the pinning of vortices to the nuclear lattice produces metastable rotational states [7]. Post-glitch response is then naturally attributed to relaxation of the inner-crust superfluid (*e.g.*, [8]). Ignoring the natal magnetic field of the NS, Ref. [9] argued that magnetic flux tubes of the outer core could form clusters around vortices, which could increase the coupling time to observable timescales of days or longer.

In this Letter, I revisit the issue of core coupling considering an essential effect of type II superconductivity in the outer core, that of the *pinning* of vortices to magnetic flux tubes associated with the star's natal magnetic field. As the protons condensed when the star was young, the very high electrical conductivity of the relativistically-degenerate electrons prevented Meissner expulsion of the core magnetic field [10], and the field formed a dense tangle of magnetic flux tubes with which vortices interact. This primarily magnetic interaction pins the vortices to the magnetic tangle which is frozen to the superconducting fluid. I show that the coupling time between the neutron and charged fluids is days to years, so that *glitch recovery involves the response of the supra-nuclear-density matter of the outer core*, where pinning occurs through a very different mechanism than in the inner crust. The problem addressed here was also considered by Ref. [11], who concluded that the relaxation time is << 1 d; the analysis given here gives a much longer longer timescale.

Vortex drag. The equation of motion of a straight vortex moving at velocity $\boldsymbol{v}$ with respect to the proton-electron fluid is given by balance between the Magnus force on the vortex and the drag force. In the rest frame of the charged component:

$$\rho_s \boldsymbol{\kappa} \times (\boldsymbol{v} - \boldsymbol{v_s}) - \eta \boldsymbol{v} = 0, \tag{1}$$

where $\boldsymbol{v_s}$ is the flow velocity of the ambient superfluid, ρ_s is the superfluid mass density, $\kappa = h/2m_n$ is the vorticity quantum where m_n is the neutron mass, $\boldsymbol{\kappa}$ is directed along the vortex, and η is the drag coefficient. In cylindrical coordinates (r, ϕ, z), with z and $\boldsymbol{\kappa}$ along the rotation axis, the vortex velocity is given by [12]

$$\boldsymbol{v} = v_s \left(\frac{1}{2} \sin 2\theta_d \, \hat{r} + \cos^2 \theta_d \, \hat{\phi} \right), \tag{2}$$

where the *dissipation angle* θ_d is given by

$$\tan \theta_d \equiv \frac{\eta}{\rho_s \kappa}. \tag{3}$$

The vortex moves at an angle θ_d with respect to $\boldsymbol{v_s}$. Entrainment of the neutron and proton mass currents when both species are superfluid endows a vortex with a magnetic field of $B_v \sim 10^{14}$ G [4]. Electron scattering with the strongly-magnetized vortex cores gives [5]

$$\tan \theta_d = (\Omega_s \tau)^{-1} \sim 10^{-3}, \tag{4}$$

in the outer core, where Ω_s is the spin rate of the superfluid and τ is the dynamical relaxation time between

the neutron fluid and the charged fluid. While the dissipation angle is much smaller than unity, it is still large enough that the neutral and charged fluids are coupled over a timescale of minutes.

Pinning of vortices to flux tubes. The magnetic field in a flux tube is $B_\Phi \sim 10^{15}$ G [4]. The magnetic interaction energy between a vortex and a flux tube E_p is of order $B_v B_\Phi V$, where V is the overlap volume, and has been estimated to be ~ 100 MeV by a number of authors [13, 14]. The angle-averaged interaction energy for the intersection of a vortex with a flux tube is [14]

$$E_p \simeq 10^2 \left(\frac{m_p^*/m_p}{0.5}\right)^{-1/2} \left(\frac{|\delta m_p^*|/m_p}{0.5}\right) \left(\frac{x_p}{0.05}\right)^{1/2} \times \left(\frac{\rho_{14}}{4}\right)^{1/2} \text{ MeV}, \quad (5)$$

where m_p is the bare proton mass, $m_p^* \equiv m_p + \delta m_p^* \sim m_p/2$ [15] is its effective mass, $x_p \simeq 0.05$ is the proton mass fraction, and ρ_{14} is the mass density in units of 10^{14} g cm^{-3}. The range of the interaction between a vortex and flux tubes is of order the characteristic radius of a flux tube, or the London length, given by [4],

$$\Lambda_* = 50 \left(\frac{m_p^*/m_p}{0.5}\right)^{1/2} \left(\frac{x_p}{0.05}\right)^{-1/2} \left(\frac{\rho_{14}}{4}\right)^{-1/2} \text{ fm}. \quad (6)$$

The pinning force is $F_p \sim E_p/\Lambda_*$, about 1 MeV fm^{-1}. For a single vortex immersed in a tangle of flux tubes, the average length between intersections will equal the average distance between flux tubes, $l_\Phi = n_\Phi^{-1/2}$, where $n_\Phi = B/\Phi_0$ is the areal density of flux tubes, B is the average magnetic field strength, and $\Phi_0 \equiv hc/2|e|$ is the flux quantum. (This argument is unaltered if flux tubes cluster around vortices [9]). The flux tubes are very numerous:

$$n_\Phi \simeq 5 \times 10^{19}\, B_{13} \text{ cm}^{-2}, \quad (7)$$

where B_{13} is the magnetic field in units of 10^{13} G. (The relevant field is the average *internal* field of the star, which can be a factor ~ 10 larger than the dipole field). The average separation of pinning intersections along a vortex is

$$l_\Phi \simeq 1600\, B_{13}^{-1/2} \text{ fm}. \quad (8)$$

The vortices are far less numerous than the flux tubes, $\sim 10^5(\Omega_s/100 \text{ rad s}^{-1})$ cm^{-2}.

The maximum differential velocity v_c between the superfluid and the flux tube array that can be sustained by pinning follows by equating the Magnus force on a vortex segment of length l_Φ to the pinning force per vortex-flux tube junction,

$$\rho_s \kappa v_c l_\Phi = E_p/\Lambda_*, \quad (9)$$

where $\rho_s \simeq \rho$ is the mass density of the neutron superfluid. Combining eqs. [5], [6], [8], and [9], gives [14]

$$v_c \sim 3 \times 10^5 \left(\frac{x_p}{0.05}\right) \frac{|\delta m_p^*|}{m_p^*} B_{13}^{1/2} \text{ cm s}^{-1}. \quad (10)$$

As a result of pinning of vortices to flux tubes, the differential velocity v_s approaches v_c as the charged component is spun down by the electromagnetic torque, but the neutron superfluid is not. Before v_c is reached, though, vortices will move through thermal activation over the pinning barriers presented by the flux tubes.

Thermally-activated vortex creep. A vortex in the outer core spends most of its time pinned to flux tubes. Occasionally a thermal excitation frees a segment of the vortex, which then moves at the velocity given by eq. [2]. After moving a distance l_Φ, on average, the vortex will encounter a flux tube segment to which it pins. The probability $\mathcal{P}$ that a vortex segment is unpinned at any instant is

$$\mathcal{P} = \mathrm{e}^{-A/T}, \quad (11)$$

where A is the activation energy for unpinning (specified below) and T is the temperature in units of energy. The average *vortex creep velocity* is

$$\boldsymbol{v} = v_s \left(\frac{1}{2}\sin 2\theta_d\, \hat{r} + \cos^2\theta_d\, \hat{\phi}\right) \mathrm{e}^{-A/T}. \quad (12)$$

The relaxation time. Assuming axisymmetry, the equation of motion of the crust plus charges (component c) of spin rate Ω_c and moment of inertia I_c, and the superfluid (component s) of moment of inertia I_s, is given by

$$I_c\dot{\Omega}_c + \int dI_s \dot{\Omega}_s = N_{\rm ext}. \quad (13)$$

where $N_{\rm ext}$ is the external electromagnetic torque, which acts only on component c. The angular acceleration of the superfluid follows from vorticity conservation [8, 16]

$$\dot{\Omega}_s(r,t) = -\frac{1}{r}\left(2\Omega_s(r,t) + r\frac{\partial}{\partial r}\Omega_s(r,t)\right) \boldsymbol{v}\cdot\hat{r}, \quad (14)$$

and is determined by the radial component of the vortex velocity which vanishes in the limit of zero dissipation ($\theta_d = 0$). In rotational equilibrium, the crust and superfluid are both spinning down at the same rate $\dot{\Omega}_0$, so that $N_{\rm ext} = I\dot{\Omega}_0$, where $I \equiv I_c + I_s$. The effects of entrainment in the core, though responsible for pinning, are not very important for the rotational dynamics [14], and so have been ignored here.

Assuming uniform superfluid rotation $\partial\Omega_s/\partial r = 0$ for simplicity, the equation of motion for the angular velocity lag $\omega \equiv \Omega_s - \Omega_c$ is

$$\dot{\omega} = \frac{I}{I_c}(|\dot{\Omega}_0| + \dot{\Omega}_s), \quad (15)$$

where

$$\dot{\Omega}_s = -\Omega_s\,\omega\sin 2\theta_d\,\mathrm{e}^{-A(\omega)/T}. \qquad (16)$$

Let the system be in spin equilibrium at $t=0$, with $\dot{\Omega}_s = \dot{\Omega}_c = \dot{\Omega}_0$ for lag ω_0. An approximate (though accurate) solution can be obtained by first expanding $A(\omega)$ about ω_0, to obtain

$$\dot{\Omega}_s \simeq \dot{\Omega}_0\,\mathrm{e}^{a(\omega-\omega_0)} \qquad a \equiv -\frac{1}{T}\frac{\partial A}{\partial\omega}\bigg|_{\omega_0}. \qquad (17)$$

Suppose a glitch occurs at $t=0$, through some instability that need not be specified for this perturbative analysis. Part of the superfluid interior spins down, delivering angular momentum to the crust and spinning it up; the lag ω decreases everywhere. Let the glitch decrease ω from ω_0 to $\omega(0+) < \omega_0$. The solution to eqs. [15]-[17] is

$$\dot{\omega} = \frac{I}{I_c}|\dot{\Omega}_0|\left[1 - \frac{1}{1+(\mathrm{e}^{t_0/\tau}-1)\mathrm{e}^{-t/\tau}}\right]. \qquad (18)$$

where

$$\tau \equiv \frac{I_c}{aI|\dot{\Omega}_0|} \qquad t_0 \equiv \frac{I_c}{I|\dot{\Omega}_0|}(\omega_0 - \omega(0+)), \qquad (19)$$

Here τ is the intrinsic relaxation time of the system, and t_0 is the *offset time*. The response of the system is delayed by t_0 before relaxing exponentially over a timescale τ. This behavior is a consequence of the non-linear dependence of the creep velocity on ω [8, 16].

The activation energy A for a parabolic pinning force is [17]:

$$A(\omega) = E_p\,(1-\omega/\omega_{cr})^{3/2}, \qquad (20)$$

where $\omega_{cr} \equiv v_c/r$ is the local critical angular velocity lag for unpinning [18]. The equilibrium lag ω_0 is given by the solution to eq. [16] with $\dot{\Omega}_s = \dot{\Omega}_0$. Introducing the spin-down time $t_{sd} \equiv \Omega_c/2|\dot{\Omega}_0|$, the relaxation time from eqs. [17], [19], and [20], is

$$\tau = \frac{2T}{3|\dot{\Omega}_0|\rho_s r\kappa l_\Phi\Lambda_*}\left(\frac{I_c}{I}\right)\left(\frac{E_p}{T}\right)^{1/3}\times \ln^{-1/3}\left[2t_{sd}\,\omega_{cr}\sin 2\theta_d\,\right], \qquad (21)$$

where the logarithm was evaluated with $\omega \simeq \omega_{cr}$ and $\Omega_s \simeq \Omega_c$. The relaxation time increases if $E_p\,T$, or B are increased, and decreases very weakly with the dissipation angle θ_d.

Taking $\rho_s = 4\times10^{14}$ g cm^{-3} and $r = 10^6$ cm, the result in terms of fiducial values is

$$\tau \simeq 15\left(\frac{I_c}{I}\right)\left(\frac{t_{sd}}{10^4\ \mathrm{yr}}\right)\left(\frac{\Omega_c}{100\ \ \mathrm{rad\ s^{-1}}}\right)^{-1}\left(\frac{\Lambda_*}{50\ \mathrm{fm}}\right)^{-1}\times \left(\frac{T}{10\ \mathrm{keV}}\right)^{2/3}\left(\frac{E_p}{100\ \mathrm{MeV}}\right)^{1/3}B_{13}^{1/2}\ \mathrm{days}. \qquad (22)$$

If Ω_s is largely unperturbed by the glitch except in a localized region, the lag immediately after the glitch will be $\omega(0+) = \omega_0 - \Delta\Omega_c$, where $\Delta\Omega_c$ is the observed change in the spin rate of the crust. The offset time is then

$$t_0 = 2\,t_{sd}\frac{\Delta\Omega_c}{\Omega_c} = 7\left(\frac{t_{sd}}{10^4\ \mathrm{yr}}\right)\left(\frac{\Delta\Omega_c/\Omega_c}{10^{-6}}\right)\ \mathrm{days}. \qquad (23)$$

The total response time to the glitch is $\simeq t_0+\tau$, about 20 days for these fiducial values, and much longer for older, cooler stars. For example, for $t_{sd} = 10^5$ yr, Ω_c=10 rad s^{-1}, $B_{13}=1$, and $T=3$ keV, the relaxation time τ becomes 18 months. The dependence of τ on E_p is weak, and the relaxation time remains long even if E_p =100 MeV is a significant overestimate. In regions of the star where the superfluid spins down, the offset time can be considerably larger. Eq. [22] suggests that the coupling time in magnetars could be a century or more, but since the fields in these objects are comparable to the lower critical field for superconductivity($H_{c1} \simeq 10^{15}$ G), the pinning description used here is likely to break down.

Discussion. The chief result of this Letter is that pinning of vortices to flux tubes in the outer core decouples the neutron superfluid from the rest of the star over timescales of weeks or longer. Given the complexities involved in measuring post-glitch relaxation, detailed comparison of eq. [21] with individual glitches is perhaps not yet warranted, but the calculated response times generally agree in magnitude with observed post-glitch relaxation timescales. Since the outer core comprises much of the moment of inertia of the star, and much more than the inner-crust superfluid, I propose that *observed post-glitch relaxation represents dynamical response of the outer core.* For older pulsars, which are also cooler, the relaxation time is years. Such long relaxation times might explain the nearly step-like nature of many pulsar glitches, with little subsequent relaxation. The treatment here assumes that at least part of the star is in rotational equilibrium before the glitch. Observations show that most pulsars do not generally relax completely between glitches [1], and this fact should be considered in further theoretical analysis and modeling of post-glitch response.

The results here are at odds with those of Sidery and Alpar [11], who found $\tau \simeq 10^{-8}(T/10\ \mathrm{keV})\exp(E_p/T)$, giving $10^{-3}\,(T/10\ \mathrm{keV})$ s for their choice of $E_p = 0.1$ MeV, a very small pinning energy compared to typical estimates of 100 MeV [13, 14]. For these values, τ given by eq. [22] is ~ 2 days. The large discrepancy arises from an unphysical expression for the creep velocity used in Ref. [11]; see [19].

A statistical study of the average angular momentum transfer in glitches in the Vela and other pulsars [20] shows that glitches are driven by some component of the star that has a moment of inertia I_g that satisfies

$$I_g \geq 0.016 I_c, \qquad (24)$$

recently confirmed with more data [21–23]. The moment of inertia of the inner-crust superfluid is at least several percent of the star [24], supporting the idea that glitches arise in the inner crust. Recent work [25], however, shows that most of the neutron mass of the dripped neutrons of the inner crust will be entrained by the nuclear clusters through Bragg scattering with the lattice, an effect that occurs whether the neutrons are superfluid or not. Entrainment of the neutrons requires the free crust neutrons to account for much more than 1.6% of I_c if glitches arise in the inner crust, as pointed out in Refs. [22] and [25]. Accounting for entrainment [23], eq. [24] becomes

$$I_g > 0.08\, I_c \tag{25}$$

while the moment of inertia in the inner-crust superfluid I_{sc} is [23]

$$I_{sc} \simeq 0.04\, I, \tag{26}$$

for a NS of mass $1.4 M_\odot$ and $R = 13$ km. If $I \simeq I_c$, due to tight coupling between the crust and the core superfluid, these considerations appear to rule out the inner crust superfluid as the origin of glitches. Entrainment causes most of the neutron fluid to spin down with the crust, and the unentrained conduction neutrons cannot accumulate angular momentum at a sufficient rate to drive large glitches.

As shown here, however, the effective moment of inertia of the crust I_c represents less of the star than previously thought, since the outer core is decoupled from the crust over observable timescales. Taking, for example, $I_c = I/2$, eqs. [25] and [26] can both be satisfied, and an inner-crust origin of glitches remains viable. Glitches might still arise in the core, but an inner-crust origin is not excluded as argued in Refs. [22] and [23]. Calculations of the moment of inertia of the type II region of the core are needed to resolve this issue. In any case, the long coupling times found in this Letter indicate that the core is a viable source of angular momentum for glitches.

At higher densities in core, the proton condensate is expected to change from type II to type I, in which case the magnetic flux will be confined to mesoscopic regions, probably with a complex geometry. Here vortex pinning might be negligible for particular magnetic field geometries [26]. On the other hand, in a type I superconductor with a complicated field structure, the gradients in the magnetic forces could still be large, resulting in vortex pinning that might be nearly as effective as that in the outer core [27], leaving almost the entire core decoupled by pinning of vortices to magnetic structures. If pinning is effective throughout the core, the effective moment of inertia of the crust plus charges could be as small as $\simeq x_p I \simeq 0.05 I$, and the angular momentum requirements of glitches become considerably easier to satisfy.

More work is needed to understand the extent to which the *inner* core fluid is coupled to the crust. Finally, I note that vortex creep in the outer core is unstable [14], which could affect the coupling time calculated here.

I am grateful to N. Chamel for very useful discussions about entrainment in the crust. This work was supported by NSF Award AST-1211391 and NASA Award NNX12AF88G.

* link@physics.montana.edu;

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